A MODEL FOR ANALYSIS OF TRANSPORTATION SYSTEMS

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ABSTRACT

Transportation systems in many nations of the world may not be adequate to support large scale combat operations against insurgent forces. The loss of capacity in a system due to insurgent action is related to certain parameters that characterize a transportation arc. These (possibly) reduced capacities become input parameters for the minimum cost flow solution of the transportation problem. Suggestions are made for determining the relationships between the parameters and the loss of capacity of the system due to insurgency. Possible areas for future study and other methods of approach are also discussed.

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1. Introduction.

Background

The coming decade will see revolutionary war occurring in various parts of the World.[1] The"newly-emerging nations" or the underdeveloped countries are those most susceptible to this type of internal stress.[2]

Examination of the histories of such revolutions as those in Algeria [7], Cuba [11], and Greece [4] shows that during the intitial phases of the war in a country, insurgents who were indigenous to that region took up arms against the legal government. They conducted a campaing of terror, oppression, sabotage, arson, and muder in order to advance their cause.

Mao Tse-Tung has stated that the revolutionist must be "... against the mere routing of the enemy, and for a war of annihilation."[3] Annihilation of the enemy is a means by which the insurgents reach their ultimate objective, becoming the de jure as well as the de facto government. This may be done by killing the government's representatives and their supporters, forcing them into exile, or, preferably, converting them to the revolutionary cause.

Che Guevara has commented on the ultimate objective of a revolutionary movement. In his book on the Cuban revolution, he states:

[1] Griffith, S.B., O. E. Chubb, and P. Durdin, "Red China," Marine Corps Gazette, 49:10-33, October, 1965.

[2] Rostow, W.W., "Guerrilla Warfare in Underdeveloped Areas," Marine Corps Gazette, 32:1-46, January, 1962.

[3] Mao Tse-Tung, Selected Works (New York: International Publishers, 1954), I, pp. 116-118.

> Final liberation comes only with the total systematic break-up of the enemy army and all institutions that supported the old regime.[4]

The legally constituted government, in its attempt to supress an insurrection, often finds it necessary to deploy its military forces throughout the country. Other nations may be requested to provide assistance in supporting the deployed government forces, and to help maintain order. The United States provided such assistance to Greece in 1946, to the Philippines in 1948, to Lebanon in 1958, and now provides assistance to Thailand and Viet Nam. Britain provided assistance to Malaya during a ten year counterinsurgency campaign in the 1950's.

To destroy government forces, the insurgents may attempt to gain tactical superiority, either by surprise or by concentrating forces.[5] Attacking lines of communication isolates government units, making them more vulnerable to insurgent attacks.[6]

Efficient operations of the existing modes of transportation for the logistical support of deployed forces, together with the introduction of new equipment, is an important factor in determining the government's ability to conduct a successful counterinsurgency operation.[7] Before conducting

[4]Che Guevara on Guerrilla Warfare (New York: Frederick A. Praeger, 1961), p. 61.

[5]Mao Tse Tung, op. cit., pp. 116-128; Valeriano, N.D. and C.T.R. Bohannan, Counterguerrilla Operations (New York: Frederick A. Praeger, 1964).

[6]Bayo, Alberto, 150 Questions for a Guerrilla (Denver: Cypress Printing Company, 1963), p. 30.

[7]Galula, David, Counterinsurgency Warfare (New York: Frederick A. Praeger, 1964), pp. 107-123.

such operations, the government may need to know the capacities of the various available modes of transportation, and their vulnerability to attack. The weak points in the system should be recognized, and measures to correct the deficiencies should be determined.

Supplies that are shipped within a transportation system may be considered to fall into one of three categories:

(1) Assault - materials transported with troops for an attack or evacuation;

(2) Emergency - unscheduled resupply, usually consisting of food, medicine, or ammunition;

(3) Regular - scheduled resupply of those items consumed during ordinary operations.

Assault and emergency supply often use transportation units organic to the assault units or to their supporting units. Regular scheduled resupply possibly could use a major portion of those transportation facilities available to the public, such as highways, railroads, and canals. In some cases, additional military units may be added to augment the public facilities. It may be considered that the largest bulk of the materials required for regular resupply of units in the field could be shipped by public transportation facilities. For this reason, only category (3), regular resupply, is considered in this investigation.

In many nations that face the threat of insurgency, an adequate public transportation system for civilian use does not exist. The additional burden of supporting military units, providing for their regular resupply,

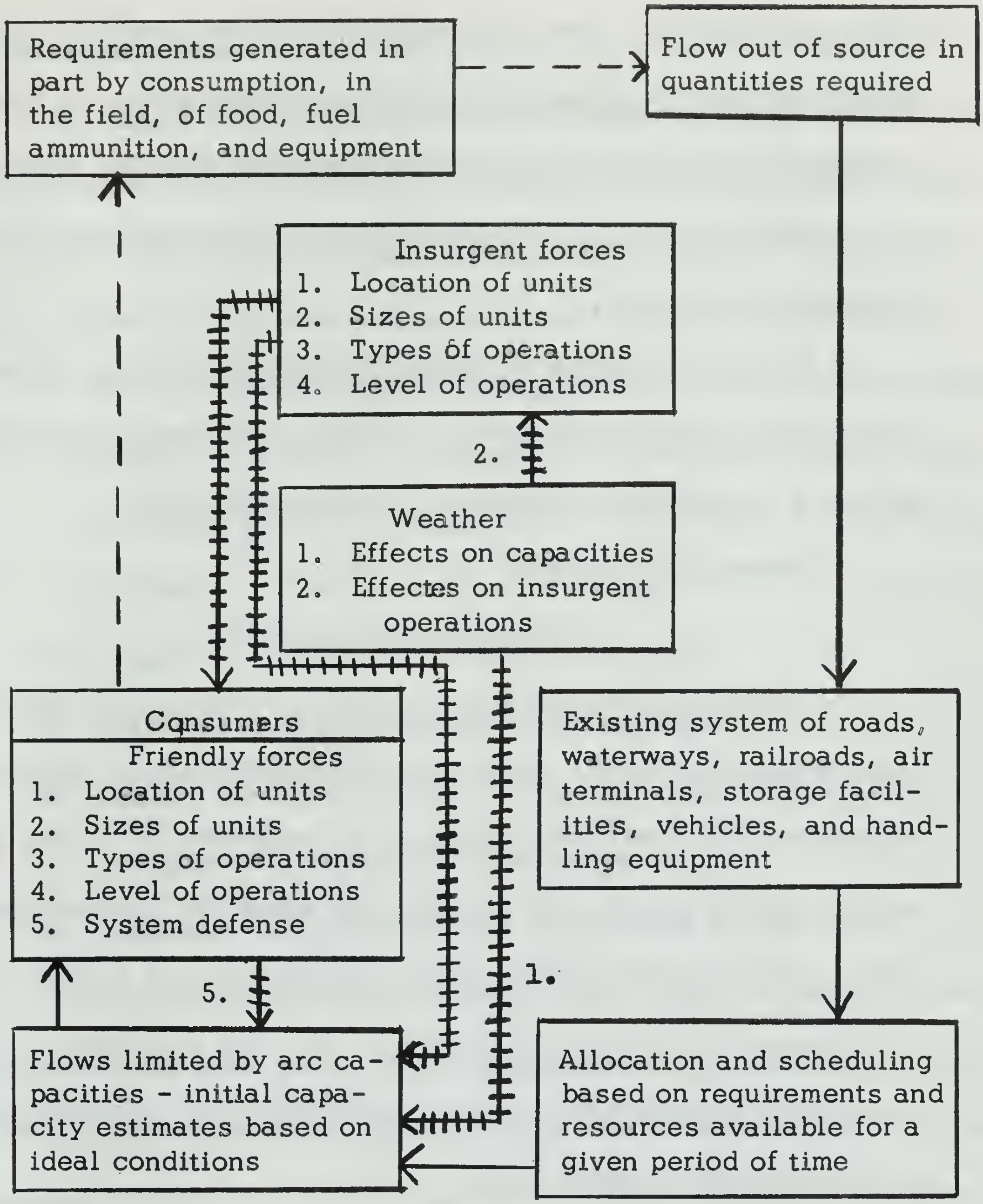
would be impossible to carry in most cases. Both military and civilians should have access to a transportation system that provides for adequate two-way movement of necessary materials.

Development of a mathematical model may facilitate the design of a transportation system. The use of this model would permit the examination of effects caused by insurgency and natural interference, the relative value of different transportation means, analysis of trade-offs, and variations in cost effectiveness. Various mixes of transportation modes could be simulated to examine their effect on flow in the system. The designed system may be compared with existing systems for the purpose of determining procurement and construction needs.

The Problem

The purpose of this investigation is to develop and define a model that can be used to analyse transportation systems in various geographic locations where the system is subject to disturbances caused by insurgent actions, terrain, weather, vehicle availability and civilian traffic. Since little is known about the effects of insurgent actions upon a transportation system, the proposal of a method by which to estimate such effects becomes part of the problem. Figure 1 is a block diagram that shows the interactions of the disturbance factors with the operating system. These interactions are discussed in section 2.

The model is developed in section 2. The development begins with a discussion of the parameters of the transportation problem and some pre-



KEY:

Information — — — —
Flow ————
Effects ++++++++

FIGURE 1

DIAGRAM OF INTERACTIONS OF DISTURBANCE FACTORS WITH THE OPERATING SYSTEM

liminary definitions. Then, following a list of the system's assumed characteristics, the proposal of a method for estimating the effects of insurgent actions is presented along with an outline of data required. Finally, the selection of a representative model for the analysis of transportation systems is discussed.

Section 3 contains the discussion of the use of the model. Results, conclusions and recommendations for futher investigation are presented in section 4. A selected bibliography concludes the paper.

2. Design of the model.

Discussion of the Variables

Optimality, as viewed in this study, implies minimizing the cost of achieving objectives. A transportation system may be defined as the aggregate of all facilities available to an organization, or to a group of people, for the movement of material and personnel from one place to another. For the purpose of this study, the objective of a transportation system is to move material and/or personnel at the rates demanded. It follows that the optimum transportation system will be usch that this movement may be accomplished at minimum cost.

The demands for movement, called requirements in this study, are stated as amounts required per unit time. Requirements are not necessarily constant. They may change during any period of time.

Requirements are assumed to be generated by consumption of food, ammunition, fuel, and equipment in the field. The amounts required per unit time depend in part on the number of troops to be supplied and on the

the type of operations they perform.

Time enters the problem through the statement of requirements. But, if time is discretized by choosing a standard unit such as a week, all requirements can be stated in terms of such a standard unit. Once the unit is chosen, time is contained implicitly in a statement of requirements.

The following definitions may be applied to a transportation system:

(1) Flow - the amount of material and/or personnel moved between any two points in a transportation system during a given period of time;

(2) Node - an origin or a destination of flow;

(3) Arc - the route of flow between two nodes;

(4) Mode - type of arc, road, rail, or air. Each mode constitutes a distinct arc between any two nodes;

(5) Capacity - an upper limit on flow through a given arc.

The movement (flow) of goods through the system is limited by various characteristics of the system. The capacity of an arc under ideal conditions is determined by characteristics which can be measured directly or estimated with reasonable accuracy. Other factors, not characteristic to the arcs, tend to reduce this ideal capacity. For example, insurgent troops may block a road or blow up a bridge. These "other" factors are the ones considered in this study. They are discussed in the next section.

Generally, the costs of operating a transportation system may be determined on the basis of requirements, system and resource characteristics, allocation and scheduling. A cost per unit flow may be assigned to each arc.

Optimal allocation and scheduling are prerequisites of the minimum cost solution to a transportation problem.

The system is assumed to have the following characteristics:

(1) Requirements, stated in tons per unit time, are predetermined inputs to the problem. Requirements at terminals or some portion of them may be considered to be lower bounds on flow;

(2) Flow may be considered to be distributed uniformly over time. For example, if it takes a convoy of trucks five days to travel an arc while transporting thirty-five tons of material, the flow may be assumed to be five tons per day. This may imply the existence of a stock level at the receiving terminal, but such an implication seems reasonable and does not effect the solution.

(3) Ideal (initial) capacities may be estimated with reasonable accuracy. They are based on the restrictions imposed by vehicle and cargo characteristics, terrain, traffic, and allocation and scheduling decisions.

(4) The most significant factors that reduce these capacities are weather and insurgent operations.

(5) Weather significantly affects only air transportation in the short run. In this case, weather may be predicted so as to allow adjustment of the system. Long run weather effects on surface transport may be included in the initial capacities for any particular period of time.

(6) The costs of constructing the existing transportation system are

sunk costs. Operating costs assigned to an arc include the cost of defending the arc.

(7) Costs vary linearly with flow in the range considered. That is, the cost per unit flow on any arc is constant during the time period covered by a solution. Even if costs do not vary linearly, a straight line approximation over a small range of flow seems feasible.

(8) The sum of flows out of any node is equal to the sum of flows into it. There are no gains or losses. The effects of loss of supplies to attacking insurgent forces may be examined parametrically by increasing costs and reducing capacities on particular arcs.

Determination of Constraints on Flow

Insurgents may attempt to occupy and control a base area that is relatively free from attack by government forces.[1] In his writings, Mao mentions the importance of base areas to provide rest, training, and medical facilities and to manufacture basic necessities for an expanding guerrilla army.[2]

The base area of the insurgent forces may be within the country they are attempting to control. The revolutionists in Cuba and the Philippines had such bases. The base area could lie outside the boundaries, in a nation

[1] Ney, Virgil, Notes on Guerrilla War (Washington: Command Publications, 1961), pp. 8-9.

[2] Mao Tse-Tung, op. cit., pp. 176-183.

whose government is sympathetic with the insurgents' cause. Yugoslavia provided such a base area for the Greek insurgents,[3] and the F.L.N. required Tunisian territory to support their operations.[4]

Operating in a contested area, the insurgents usually attempt to extend their control to the government base area. In some cases, the whole country may be contested. Areas of the region may be secure in varying degrees against the encroachment of control by one side or the other.

The Republic of Viet Nam is an example of an entire country that is contested. The insurgents' former base areas, such as the "Iron Traingle" or the central highlands, are now contested due to the striking ability of the air mobile units and the air attacks by B-52 bombers. Similarly, no area under nominal government control is free from the danger of a terrorist bomb or a mortar attack. Therefore, the transportation arcs required by government forces to move necessary supplies are subject to attack by the insurgents in the contested areas.

The reasons for attacking transportation arcs are not stated explicity in the writings of many of the theoreticians and practitioners of guerrilla warfare. However, a few of the implied reasons may be deduced [1], [2], [9]. Some of these reasons are to:

(1) Prevent the movement of government troops;

[3]Murray, J. C., "The Anti-Bandit War," Marine Corps Gazette, 38:1-15, January, 1954.

[4]Paret, P. and J. W. Shy, "Guerrilla War and U. S. Military Policy," Marine Corps Gazette, 46:1-29, January, 1962.

(2) Capture needed supplies;

(3) Kill government troops as part of an attrition campaign;

(4) Harass government forces, causing them to spread their defenses;

(5) Cause labor and materials to be directed from the counterinsurgency effort to maintaining the transportation system;

(6) Hinder the movement of civilian goods and products in an attempt to disrupt the nation's economy;

(7) Demonstrate active opposition to the government, and to encourage others to join the insurgent forces;

(8) Discredit the government, showing that it has no power to protect its citizens.

A survey of the literature indicated that there are parameters that may affect the capacity of an arc.

These parameters are related to inherent characteristics of the arc, the requirements for use of the arc, and allocation of resources by friendly and enemy forces. These parameters include:

(1) Importance of the arc to the government;

(2) Vulnerability of the arc to damage by the insurgents;

(3) Accessibility of the arc to the insurgents;

(4) Defensive forces assigned to that arc.

Measurement of these characteristics could lead to determining the effect they have on the loss of capacity due to insurgent attack. Data on these four factors, collected and analyzed with respect to loss of capacity, may make it possible to determine definite relationships. This may permit

prediction of the loss of capacity in a transportation system, or estimation of defensive forces required to keep losses at a given level. Therefore, Importance, Vulnerability, and Accessibility are defined and some relationships are proposed.

It is proposed that a measure of the importance I of an arc ij be proportional to the quantity of goods that are required to be shipped through the arc. Consider node j with two arcs ij and kj into j. Suppose that j has requirements Q, and this must be shipped through arcs ij and kj. The arcs have capacities M_{ij} and M_{kj}, with shipping costs C_{ij} and C_{kj}. If $C_{kj} < C_{ij}$ and $M_{kj} + M_{ij} > Q$, the one would assign arc kj to carry as much as it can, i.e. M_{kj}. The rest of the material required at j must be shipped through arc ij. Then the required flow through arc ij is $Q_{ij} = Q - M_{kj}$, and the importance of arc ij could be written as $I_{ij} = k_1 + k_2 Q_{ij}$, where k_1 and k_2 are non-negative constants.

The vulnerability of an arc is a measure of its susceptibility to damage by the means available to the insurgents. To assign a value to vulnerability, several factors are considered. Resources available to the insurgents may dictate a particular type of target, or limit the damage done in an attack. The number of bridges, tunnels, curves, locks, or other easily damaged sections of the arc seem to give an indication of its vulnerability. The government's ability to repair the arc once it is damaged is also considered. A possible measure for vulnerability is proposed to be $V_{ij} = m_1 + m_2 B_{ij}$, where m_1 and m_2 are non-negative constants, and B_{ij} is the maximum number of man-hours required to repair p percent damage to the n complex

sections of the arc if they were damaged simultaneously.

As an example of vulnerability, consider the arc ij with two bridges, a tunnel, and three hairpin curves. Assume that the insurgents have the capability of attacking any three of the vulnerable sections simultaneously, and inflicting 50% damage. In this case, n = 3, and p = .50. If it takes 250 man-hours to repair any curve if it is 50% damaged, 1200 man-hours to repair the tunnel, and 750 man-hours to repair either bridge, then

$$B_{ij} = \text{max of any 3 } (250, 250, 250, 1200, 750, 750)$$

$$B_{ij} = 250 + 1200 + 750 = 2200 \text{ man-hours}$$

The measure $V_{ij} = m_1 + m_2 B_{ij}$ implies that an arc increases in vulnerability with an increasing number of sensitive sections. If the repair crews can work on several damaged sections simultaneously, then B_{ij} would be less than if they had to be repaired in order. If n becomes large, then the increase of B_{ij} may be non-linear. However, it is assumed that in the range of interest, the linear relation will be an adequate approximation.

It is reasonable to expect that the vulnerability of an air arc, and its accessibility to insurgents might approach zero. An exception might be the situation where the insurgents are able, by using small arms fire, to force aircraft to fly around an area. Since aircraft must operate from airfields, it may be desirable to measure the accessibility of the nodes instead.

An arc must be accessible to the insurgents before they can carry out an attack. This includes the amount of cover available near the arc as well as the number of men that the insurgents have available for the attack. A proposed measure of accessibility is $A_{ij} = n_1 + n_2 E_{ij}$, where n_1 and n_2 are

non-negative constants, and E_{ij} is the estimated number of enemy troops within some distance of the arc ij. This relationship indicates that the accessibility of an arc increases when insurgents in the area increase. Although this may not be an exact linear relationship, it is assumed that in the range of interest the linear relationship is a close approximation.

Another important factor in determining the loss on an arc is the number, type, and quality of forces assigned to protect the arc. The defensive forces may be used in static positions, and their surveillance range may be increased by the use of acoustic, visual, or other types of detectors. Personnel may be used as a mobile defense, actively patrolling the area near the arc. Some combination of these two may be used.

The importance, vulnerability, accessibility, and level of insurgent activity may be assumed to determine the defensive forces assigned to protect an arc. The defense D is then:

$D_{ij} = f(I_{ij}, V_{ij}, A_{ij}, S_{ij})$, where S is a measure of the level of insurgent activity on arc ij.

The measure for D_{ij} should take into account that some defenses are more effective than others. Certain mixes of men and equipment may prove to be more effective than others in preventing insurgent attacks.

The capacity of arc ij, M_{ij}, is the limit set by the physical characteristics and vehicle allocation on the arc as previously defined. Insurgent activity may establish a new capacity, X_{ij}, such that $X_{ij} \leq M_{ij}$. The loss in capacity is defined to be $L_{ij} = M_{ij} - X_{ij}$. Since the maximum loss cannot be greater than the capacity of the arc, $\max L_{ij} = M_{ij}$.

The loss in capacity, L_{ij}, is assumed to vary inversely with the defensive forces. The relationship $L \propto \frac{1}{D}$ also depends indirectly upon I, V, A, and i. Since $D = f(I, V, A, S)$, then $L = g(I, V, A, S, D)$.

Data to determine if any dependence actually does exist and to futher develop these relationships, should be collected on a particular arc ij, and be base upon some time interval, such as a week.

The measures for I, V, and A are used for illustration only, Other, and possibly better, measures for those characteristics may be found in the analysis of the data.

The characteristics that are of interest are listed below. Included is suggested data that could be collected to determine a measure of that characteristic.

(1) Importance

(a) Number of tons of material moved over an arc.

(b) Maximum number of tons of material that could be moved under ideal conditions, M_{ij}.

(c) Dollar cost per ton to ship goods over an arc.

(d) Number of troops assigned to defend an arc.

(e) Dollar cost to defend an arc.

(2) Vulnerability

(a) Number of each type of critical sections.

(b) Time required to repair each type of critical section.

(c) Location of repair units.

(d) Location of static defenses.

(e) Number of patrols, area patrolled, and frequency of patrols.

(f) Capability of repair units.

(g) Estimated resources of the insurgents, including men and material.

(3) Accessibility

(a) Type of terrain near the arc.

(b) Location of insurgent base area.

(c) Number of insurgents within two days march of the arc.

Incident reports following an actual insurgent attack on an arc may provide more information on the proposed relationships. Data taken from the incident reports may include:

(1) Time, date and location of the attack.

(2) Weather conditions before and during the attack.

(3) Damage done, and estimated time to repair.

(4) Location of defensive forces at the time.

(5) Estimate of number of insurgents taking part.

(6) Estimate of where insurgents came from and where they went upon completing the attack.

Upon completion of repairs to the damage done by the attack, futher information should be obtained to complement data taken from the incident report. These data would include:

(1) Actual number of man-hours required to repair damage.

(2) Number of hours damaged section closed to traffic.

(3) Number of hours arc was capable of handling reduced traffic.

(4) Amount of material shipped and arc used to send supplies to the terminal of the damaged arc while repairs are being made.

The data collected on an arc while in a "steady state" condition, and immediately following an incident, together with an assessment of the repaired damage may result in the verification of the proposed relationships.

Selection of the Representative Model

Once the (possibly) reduced capacities have been determined, the problem is to find the minimum cost flow solution of the transportation problem. The input parameters are requirements, capacities, and costs. Since none of these are really determined with infinite precision, the solution may be only a close approximation to the optimal solution. It may be possible to determine the input parameters with great accuracy, but the cost of doing it would probably be prohibitive. If the cost-flow relationship is found to be curvilinear, as solution for a large system would be complicated considerably. In the interest of efficiency in the use of time and resources the simplifying assumption of linearity in the cost-flow relationship is made. This provides a linear function for minimization, which implies constant returns to scale in the economic sense. Constant returns to scale means output is directly proportional to input. And since capacities and requirements are respectively one-dimensional upper and lower bounds on flow, the constraints on the flow variables are stated in terms of linear inequalities. The linear relationships permit the formulation of this problem as a linear program.

The part of linear programming theory called network flows seems

particularly applicable to transportation problems. The network flow model using the "out-of-kilter" method of solution was chosen for several reasons. First, Fulkerson's out-of-kilter algorithm 5 for the solution of network flow problems allows the use of lower bounds (requirements), as well as upper bounds (capacities), on flow. Second, the out-of-kilter method employs an iterative procedure that is amenable to programming for computer solutions. The use of computers permits extensive parametric studies and sensitivity analysis. The parametric studies and sensitivity analysis may be accomplished with a computer simulation which allows repeated solutions while varying the input parameters. Once an optimal solution has been found with one set of input parameters, it may be used as the starting point for the next computer run with changed parameters. This saves computer operation time. Third, the method leads to a minimum cost flow solution without a non-negativity restriction on the cost coefficients. Fourth, for observers who may not be familiar with the theroy, graphical representation tends to make the complex interactions within a system more understandable. Finally, in a finite number of steps, the algorithm yields either the minimum cost solution or the conclusion that no feasible flow exists for the current state of the system.

Infeasible flows may be recognized as the existence of requirements at a terminal that are greater than any possible flow into that terminal. Such occurrences suggest adjustment of the system through changes in allocation, scheduling or even in the inherent characteristics of the system. As will be seen in the next section, the out-of-kilter algorithm determines the locations

and amounts of deficiencies in the system in terms of flows.

3. Use of the Model.

This section describes the process of representing the transportation system as a network flow problem and solving the problem by means of the out-of-kilter algorithm.

The network may be formulated according to the following procedure. Represent the system as a network with distinct arcs and nodes for each mode. Add fictitious source and sink nodes. Arcs from the source to the actual origin of system flow are assumed to have infinite capacities, zero costs, and zero lower bounds. Arcs from the nodes at which requirements exist to the sink are assumed to have infinite capacities, zero costs, and lower bounds equal to the requirements at those nodes. Add an arc from sink to source to make the flow into the system equal to the flow out of the system, and satisfy the material balance requirements. This circulation arc has requirements, capacities and flow all equal to the total flow through the network which for a feasible solution, must be equal to total requirements.

The following is the formulation of the linear program.

The Primal Problem:

$$\text{Minimize} \sum_{ij} c_{ij}\, x_{ij}$$

Subject to $\mathbb{I}X = 0$

and $L \leq X \leq M$, where

$$X = \begin{bmatrix} x_{01} \\ x_{n-1,n} \\ x_{n0} \end{bmatrix}, \quad L = \begin{bmatrix} l_{01} \\ l_{n-1,n} \\ l_{n0} \end{bmatrix}, \quad M = \begin{bmatrix} m_{01} \\ m_{n-1,n} \\ m_{n0} \end{bmatrix}$$

c_{ij} is the cost per unit flow on arc ij.

x_{ij} is the amount of flow on arc ij.

I is the node-arc incidence matrix formed as follows. Label each node numerically. List the nodes vertically and the arcs, such that arc ij begins at node i and ends at node j, horizontally. For each node-arc combination, place in the matrix a (+ 1) for flow out of the node, and a (- 1) for flow into the node, and a zero if the arc neither begins nor ends at the particular node.

l_{ij} is the lower bound on flow through arc ij. It is equal to requirements at node i on those arcs i,n joining the nodes at which requirements exist to the sink, n. l_{ij} is equal to zero on all other arcs.

m_{ij} is the upper bound on flow through arc ij. These upper bounds are the "reduced" capacities for each solution run.

The algorithm is an efficient combination of the Primal-Dual and Simplex methods for solving linear programs. It maximizes the Dual while minimizing the Primal such that when the Primal and Dual are equal, the solution is optimal.

The procedure is based on the assumption that both the primal and dual solutions exist, and on the following theorem due to Dantzig [3].

> "For optimal feasible solution of the primal and dual systems, whenever slack occurs in the k^{th} relation of either system, the k^{th} variable of its dual vanishes; if the k^{th} variable is positive in either system, the k^{th} relation of its dual is equality."

This theorem is the basis for the complementary slackness conditions, which are defined by the relations of states (a), (b), and (c) presented in the next paragraph. When the complementary slackness conditions are satisifed for every arc in the system, the solution is optimal.

An arc may be in any one of nine possible states. These are:

(a) $V_i - V_j < c_{ij}, \; x_{ij} = l_{ij}$

(b) $V_i - V_j = c_{ij}, \; l_{ij} \leq x_{ij} \leq m_{ij}$ in Kilter

(c) $V_i - V_j > c_{ij}, \; x_{ij} = m_{ij}$

(a_1) $V_i - V_j < c_{ij}, \; x_{ij} < l_{ij}$

(b_1) $V_i - V_j = c_{ij}, \; x_{ij} < l_{ij}$

(c_1) $V_i - V_j > c_{ij}, \; x_{ij} < m_{ij}$

out of Kilter

(a_2) $V_i - V_j < c_{ij}, \; x_{ij} > l_{ij}$

(b_2) $V_i - V_j = c_{ij}, \; x_{ij} > l_{ij}$

(c_2) $V_i - V_j > c_{ij}, \; x_{ij} > m_{ij}$

where the V_i are the dual variables of the $IX = 0$ relations. They are assigned to the nodes and, once an optimal feasible solution is reached, each V_i represents the minimum cost of moving the next unit of flow from node i to the sink. The $V_i - V_j$ represent this cost for arc ij. The other variables are as defined in the linear program.

An arc in state (a), (b), or (c) satisfies the complementary slackness conditions and therefore is in kilter. An arc in any of the other states does not satisfy the conditions and is out of kilter. This algorithm seeks to bring all arcs into kilter; that is, onto their complementary slackness diagrams.

The diagrams are generated by the complementary slackness conditions applicable to each arc. Figure 2 is a general example of a complementary slackness diagram showing all the possible states of an arc. The diagram shows the relationships between flow, the dual costs, assigned costs, and the upper and lower limits on flow for each arc.

Kilter numbers are defined such that zero implies that an arc is in kilter. States (a), (b), and (c) have kilter numbers of zero. For the other possible states, the kilter numbers are defined as follows:

(a_1) or (b_1) : $l_{ij} - x_{ij}$

(b_2) or (c_2) : $x_{ij} - m_{ij}$

(c_1) : $(V_i - V_j - C_{ij})(m_{ij} - x_{ij})$

(a_2) : $(V_i - V_j - C_{ij})(l_{ij} - x_{ij})$

These numbers are all positive. Kilter numbers for (a_1), (b_1), and (c_2) measure infeasibility while those for (a_2) and (c_1) measure the degree to which these states violate the complementary slackness conditions.

The remaining part of this section is a step by step outline of the out-of-kilter algorithm [5].

(1) Start with any circulation flow and assign any set of non-negative V_i to the nodes. Try to find an out-of-kilter arc - one not in any of the states (a), (b), and (c). If no out-of-kilter arcs can be found, the problem has been solved and the solution is optimal. Otherwise, go to step (2).

(2) Determine which out-of-kilter case applies. If (a_1) applies, then try to increase flow in arc ij to l_{ij} through loop flow. Begin

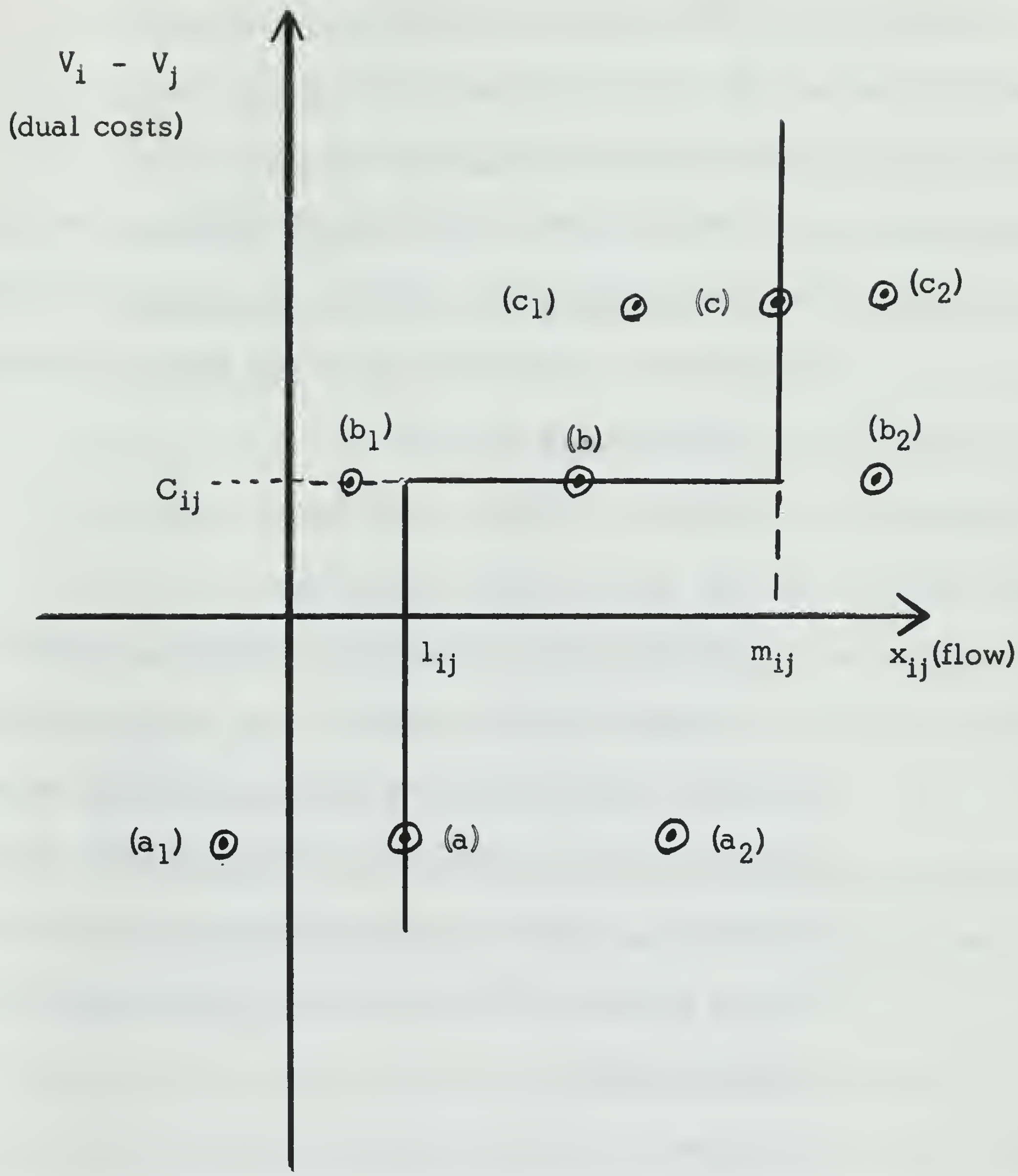


FIGURE 2

AN EXAMPLE OF A COMPLEMENTARY SLACKNESS DIAGRAM SHOWING THE POSSIBLE STATES OF AN ARC

by labelling node j with $(i/l_{ij} - x_{ij}/+)$.

(i) If node r is labeled $(k/F_r/\pm)$ and mode s is unlabelled, then if rs is an arc such that

(1) $V_r - V_s < C_{rs}$, $x_{rs} < l_{rs}$

(2) $V_r - V_s \geq C_{rs}$, $x_{rs} > m_{rs}$

then label node s with $(r/F_s/+)$ where $F_s = \min(F_r,\ l_{rs} - x_{rs})$ for case (1) and $F_s = \min(F_r,\ m_{rs} - x_{rs})$ for case (2)

(ii) If node r is labelled $(k/F_r/\pm)$ and node s is unlabelled, then if sr is an arc such that

(1) $V_s - V_r \leq C_{sr}$, $x_{sr} > l_{sr}$

(2) $V_s - V_r > C_{sr}$, $x_{sr} > m_{sr}$

then label node s with $(r/F_s/-)$ where $F_s = \min(F_r,\ x_{sr} - l_{sr})$ for case (1) and $F_s = \min(F_r,\ x_{sr} - m_{sr})$ for case (2).

If node i of the out-of-kilter arc can be labelled, send through the appropriate flow and adjust the flows in all other arcs of the loop. Of node i cannot be labelled, then a cut set $(S, \bar{S})$ results where S is the set of labelled nodes and $\bar{S}$ is the set of unlabelled nodes.

Define

$$\theta = \min \left[\min_{\substack{rs \in (S,\bar{S}) \\ \text{inactive and} \\ x_{rs} \leq m_{rs}}} \left\{ c_{rs} - (V_r - V_s) \right\} \ ; \ \min_{\substack{rs \in (\bar{S},S) \\ \text{hyperactive} \\ \text{and } x_{rs} \geq l_{rs}}} \left\{ V_r - V_s - C_{rs} \right\} \right]$$

and then set $V'_r = V_r \; \theta$ for all $r \in S$

$V'_r = V_r$ for all $r \in \bar{S}$

Note that θ determination includes arc ij. If arc ij is in kilter after the V_r's are changed, look for other out-of-kilter arcs and repeat the process of step (2). If arc ij is still out-of-kilter, continue to try to lable node i. If it is impossible to label node i, then stop; there is no feasible circulation flow and adjustments must be made to the system.

If (b_1) or (c_1) applies, begin with a label of ($i/m_{ij} - x_{ij}/+$) on node j and then proceed as for state (a_1). Here we try to increase flow in arc ij to m_{ij}. If (a_2) or (b_2) applies, then the labelling process starts at node i, instead of node j, and the label on node i is ($j/x_{ij} - l_{ij}/-$). The rest of the procedure is the same as for (a_1). Here we try to reduce the flow in arc ij to l_{ij}.

If (c_2) applies, then proceed as for (a_2) and (b_2) except that node i is labelled ($j/x_{ij} - m_{ij}/-$). Here we try to reduce the flow in arc ij to m_{ij}.

This algorithm solves the problem in a finite number of steps or terminates with the conclusion that no feasible flow exists. But, when an infeasibility exists, the kilter number defines its amount and location. This allows adjustments toward feasibility without disturbing the rest of the system. The status of any arc is not made worse at any time during the computation. All arcs that are in kilter remain in kilter. Kilter numbers for arcs that are out-of-kilter either decrease or remain unchanged at each step. In

the special case in which the initial flow is feasible, at least one kilter number decreases at each step.

4. Conclusions and Recommendations.

This paper attempts to describe the relations between losses in a transportation system and outside forces. A proposal is made to construct a deterministic mathematical model. This model, based on the parameters importance, vulnerability, accessibility, level of insurgent activity, and defensive forces seems feasible.

This model may be used to examine the effects of insurgency, the relative value of different transportation means, analysis of trade-offs, and variations in cost effectiveness. Various mixes of transportation modes could be simulated to examine their effect on flow in the system.

Several questions must be answered before relationships can be defined. Measurements for I, V, and A are proposed. Importance I is a linear function of the quantity of material required to be shipped on an arc. Vulnerability V is a linear function of the number of sensitive sections of the arc, and accessibility A is a linear function of the number of enemy troops near the arc.

An outline of the data required for the determination of the actual relationships is presented. Analysis of these data may determine whether the measures proposed for I, V, and A are acceptable. The proportionalty constants could possibly be determined from the data, also.

The use of the <u>out-of-kilter</u> algorithm for the network flow problem seems to be applicable for the transportation system. The conditions that

must be met, the implied assumptions, and the use of the algorithm is discussed for this particular situation.

No measurements for S and D are offered. Although it is indicated that S may be some measure of the level of insurgent activity, and D may be a measure of the effects of the defensive forces, further study is needed to find suitable units of measurement.

Further analysis may indicate that one or more of the five parameters selected as having an important bearing on the determination of losses may, in fact, not effect losses at all. Other parameters may become apparent, and recognition of them would be an important result from the analysis of data.

Other methods of approach may be more profitable in further studies on this subject. Instead of a deterministic value of loss on each arc, perhaps a probability distribution function could be found that would indicate the probability that losses on an arc would exceed a given amount. Modification of the out-of-kilter algorithm may be necessary before it is used with probabilistic losses.

An approach using queueing theory may be used to describe events on an arc. Consider repair of damage to the arc as a "service" and the damage from insurgent attacks as "arrivals". Data may be used to derive a probability disturbution for arrivals. The probability of an attack, or the conditional probability of losses given an attack may be of use in planning the deployment of defensive forces or for routing of shipments.

This paper assumes that the properties of the network are invariant in some short period of time. In fact, they may not be. Insurgent attacks may cause increasing (or decreasing) losses as the revolutionary war progresses. Another approach using time as a parameter may prove valuable.

In summary, we have shown that construction of a mathematical model to describe losses in a transportation system is feasible. This model would be of value to those men responsible for the safe movement of material in those areas where insurgency threatens the transportation system.

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Transportation systems in many nations of the world may not be adequate to support large scale combat operations against insurgent forces. The loss of capacity in a system due to insurgent action is related to certain parameters that characterize a transportation arc. These (possibly) reduced capacities become input parameters for the minimum cost flow solution of the transportation problem. Suggestions are made for determining the relationships between the parameters and the loss of capacity of the system due to insurgency. Possible areas for future study and other methods of approach are also discussed.

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